MATH COMPETITION TEAM – SUMMER CHECKLIST

REMIND INSTRUCTIONS

- ☐ The first document contains instructions for joining the Remind class discussion group. At any time after 7/15/20 and before the first day of school, please follow the instructions and join the group.

SYLLABUS

- ☐ Document #2 is the course syllabus. Please read it carefully.
- ☐ Document #3 contains information on the AICE Math and AICE Further Math syllabi and papers. Please read it carefully. You will have to make some decisions on day 1 regarding which papers you wish to write in May.
- ☐ You must have a three-ring binder and loose-leaf paper the first day of class.

PRACTICE

- ☐ Document #4 contains some recreational problems. They are not directly related to the course material but are designed to get your creative problem-solving juices flowing. Have fun with them.

remind

Sign up for important updates from Mr. Dominguez.

Get information for **Everglades High School** right on your phone—not on handouts.

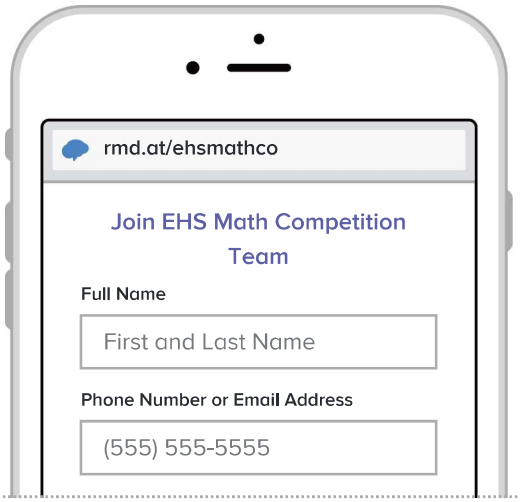
Pick a way to receive messages for **EHS Math Competition Team**:

A **If you have a smartphone, get push notifications.**

On your iPhone or Android phone, open your web browser and go to the following link:

rmd.at/ehsmathco

Follow the instructions to sign up for Remind. You'll be prompted to download the mobile app.



B **If you don't have a smartphone, get text notifications.**

Text the message @ehsmathco to the number 81010.

If you're having trouble with 81010, try texting @ehsmathco to (954) 883-9389.

** Standard text message rates apply.*

Don't have a mobile phone? Go to rmd.at/ehsmathco on a desktop computer to sign up for email notifications.

Mathematical Competition

Mathematical Competition is a high school course designed to help students prepare to successfully compete in the Florida Math League (FML) and American Mathematics Competition (AMC) math competitions. The course covers the entire high school mathematics curriculum including elementary algebra, basic geometry (including the Pythagorean Theorem, area and volume), elementary number theory (a subject not traditionally included in the curriculum), elementary probability, trigonometry, advanced algebra, and advanced geometry. Problems in FML and AMC are sometimes amenable to a calculus-based solution, but they will always have a non-calculus solution as well. The course is designed for juniors and seniors, although it may be appropriate for sophomores in some situations.

Prerequisite

AP Calculus AB with a grade of B or better, or permission of the instructor

Calculus provides the necessary mathematical maturity to be successful in this course. Also, without prior exposure to calculus, students will find it very difficult to pass the AICE Math or AICE Further Math exams.

Course Requirements

1) Students **must** be available to participate in all math competitions (AMC, Math League, RCML, Log One) **outside of regular scheduled class time** (sometimes in study hall, sometimes after school) about a dozen times over the course of the school year.
2) Juniors must write the AICE Mathematics (9709) AS exam. This will consist of Paper 1 (Pure Math 1) *plus* either Paper 4 (Mechanics) or 5 (Statistics), depending on student background and interest. This will allow students interested in repeating this course next year to register for the AICE Mathematics (9709) A level exam.
3) Seniors without credit for AICE Math must write the AICE Mathematics (9709) AS exam. This will consist of Paper 1 (Pure Math 1) and Paper 2 (Pure Math 2).
4) Seniors with credit for AICE Math must write the AICE Further Mathematics (9231) AS exam. This will consist of Paper 1 (Further Pure Math 1) *plus* Paper 3 (Further Mechanics) or 4 (Further Statistics), depending on student background and interest.

Grading Categories

- Classwork Reading, Problems, and Discussions – 20%
- Homework Problem Sets – 20%
- Competitions – 20%
- Class Exams – 40%

Primary Readings

- A. Dominguez, *Problem Solving Study Notes*
- Problem Solving Books
  - for first-year Mathematical Competition students:
    - S. Lehoczky & R. Rusczyk, *The Art of Problem Solving, Volume 1: The Basics*
    - R. Rusczyk & M. Crawford, *Intermediate Algebra*
    - D. Patrick, *Intermediate Counting and Probability*
  - for second-year Mathematical Competition students:
    - S. Lehoczky & R. Rusczyk, *The Art of Problem Solving, Volume 2: And Beyond*
    - R. Rusczyk, *Pre-Calculus*
    - D. Patrick, *Calculus*

Mathematical Competition

- Coursebooks
  - for juniors writing the AICE Mathematics (9709) AS exam:
    - S. Pemberton, *Pure Mathematics 1 Coursebook*
    - J. Dangerfield & S. Haring, *Mechanics Coursebook* **OR** D. Chalmers, *Probability & Statistics 1 Coursebook*
  - for seniors writing the AICE Mathematics (9709) AS exam:
    - S. Pemberton, *Pure Mathematics 1 & 2 Coursebook*
  - for seniors writing the AICE Further Mathematics (9231) AS exam:
    - L. McKelvey & M. Crozier, *Further Mathematics Coursebook*
- Additional readings will be assigned as needed on an ad hoc basis to accomplish class objectives.

Web Resources

- Art of Problem Solving website, https://artofproblemsolving.com/

AICE Math (9709) for Juniors

Content overview

Content section	Assessment component	Topics included
1 Pure Mathematics 1	Paper 1	1.1 Quadratics 1.2 Functions 1.3 Coordinate geometry 1.4 Circular measure 1.5 Trigonometry 1.6 Series 1.7 Differentiation 1.8 Integration
2 Pure Mathematics 2	Paper 2	2.1 Algebra 2.2 Logarithmic and exponential functions 2.3 Trigonometry 2.4 Differentiation 2.5 Integration 2.6 Numerical solution of equations
3 Pure Mathematics 3	Paper 3	3.1 Algebra 3.2 Logarithmic and exponential functios 3.3 Trigonometry 3.4 Differentiation 3.5 Integration 3.6 Numerical solution of equations 3.7 Vectors 3.8 Differential equations 3.9 Complex numbers
4 Mechanics	Paper 4	4.1 Forces and equilibrium 4.2 Kinematics of motion in a straight line 4.3 Momentum 4.4 Newton's laws of motion 4.5 Energy, work and power
5 Probability & Statistics 1	Paper 5	5.1 Representation of data 5.2 Permutations and combinations 5.3 Probability 5.4 Discrete random variables 5.5 The normal distribution
6 Probability & Statistics 2	Paper 6	6.1 The Poisson distribution 6.2 Linear combinations of random variables 6.3 Continuous random variables 6.4 Sampling and estimation 6.5 Hypothesis tests

Structure

There are six components that can be combined in specific ways (please see below for details):

Paper 1: Pure Mathematics 1
Paper 2: Pure Mathematics 2
Paper 3: Pure Mathematics 3
Paper 4: Mechanics
Paper 5: Probability & Statistics 1
Paper 6: Probability & Statistics 2

All AS Level candidates take two written papers.
All A Level candidates take four written papers.

AS Level Mathematics

The Cambridge International AS Level Mathematics qualification offers three different options:

- Pure Mathematics only (Paper 1 and Paper 2) **or**
- Pure Mathematics and Mechanics (Paper 1 and Paper 4) **or**
- Pure Mathematics and Probability & Statistics (Paper 1 and Paper 5).

Please note, the Pure Mathematics only option (Paper 1 and Paper 2) leads to an AS Level only and cannot be used as a staged route to a full A Level. Candidates who have taken Paper 1 and Paper 2 at AS Level and then wish to complete a full A Level would need to retake Paper 1 alongside three other components. These should be chosen from the specific combinations below.

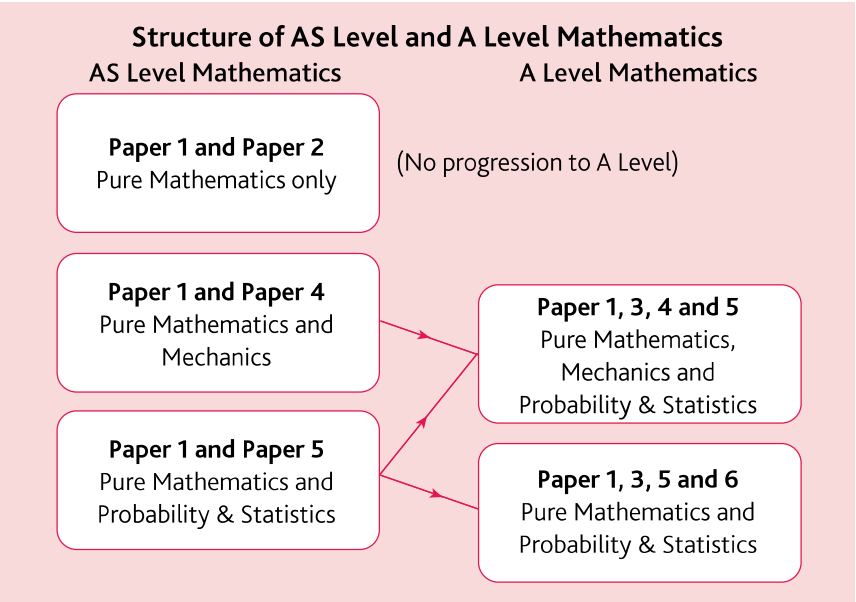
A Level Mathematics

The Cambridge International A Level Mathematics qualification offers two different options:

- Pure Mathematics, Mechanics and Probability & Statistics (Papers 1, 3, 4 and 5) **or**
- Pure Mathematics and Probability & Statistics (Papers 1, 3, 5 and 6).

Please note, it is not possible to combine Paper 4 and Paper 6. This is because Paper 6 depends on prior knowledge of the subject content for Paper 5.

See page 10 for a table showing all possible assessment routes.



Assessment overview

Paper 1

Pure Mathematics 1 1 hour 50 minutes

75 marks

10 to 12 structured questions based on the Pure Mathematics 1 subject content

Written examination

Externally assessed

60% of the AS Level

30% of the A Level

Compulsory for AS Level and A Level

Paper 2

Pure Mathematics 2 1 hour 15 minutes

50 marks

6 to 8 structured questions based on the Pure Mathematics 2 subject content

Written examination

Externally assessed

40% of the AS Level

Offered only as part of AS Level

Paper 3

Pure Mathematics 3 1 hour 50 minutes

75 marks

9 to 11 structured questions based on the Pure Mathematics 3 subject content

Written examination

Externally assessed

30% of the A Level only

Compulsory for A Level

Paper 4

Mechanics 1 hour 15 minutes

50 marks

6 to 8 structured questions based on the Mechanics subject content

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Paper 5

Probability & Statistics 1 1 hour 15 minutes

50 marks

6 to 8 structured questions based on the Probability & Statistics 1 subject content

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Compulsory for A Level

Paper 6

Probability & Statistics 2 1 hour 15 minutes

50 marks

6 to 8 structured questions based on the Probability & Statistics 2 subject content

Written examination

Externally assessed

20% of the A Level only

Offered only as part of A Level

AICE Further Math (9231) for Seniors

Content overview

Content section	Assessment component	Topics included
1 Further Pure Mathematics 1	Paper 1	1.1 Roots of polynomial equations 1.2 Rational functions and graphs 1.3 Summation of series 1.4 Matrices 1.5 Polar coordinates 1.6 Vectors 1.7 Proof by induction
2 Further Pure Mathematics 2	Paper 2	2.1 Hyperbolic functions 2.2 Matrices 2.3 Differentiation 2.4 Integration 2.5 Complex numbers 2.6 Differential equations
3 Further Mechanics	Paper 3	3.1 Motion of a projectile 3.2 Equilibrium of a rigid body 3.3 Circular motion 3.4 Hooke's law 3.5 Linear motion under a variable force 3.6 Momentum
4 Further Probability & Statistics	Paper 4	4.1 Continuous random variables 4.2 Inference using normal and *t*-distributions 4.3 χ^2-tests 4.4 Non-parametric tests 4.5 Probability generating functions

Structure

There are four components that can be combined in specific ways (please see below):
Paper 1: Further Pure Mathematics 1
Paper 2: Further Pure Mathematics 2
Paper 3: Further Mechanics
Paper 4: Further Probability & Statistics

All AS Level candidates take two written papers.
All A Level candidates take four written papers.

AS Level Further Mathematics

The Cambridge International AS Level Further Mathematics qualification offers two different options:

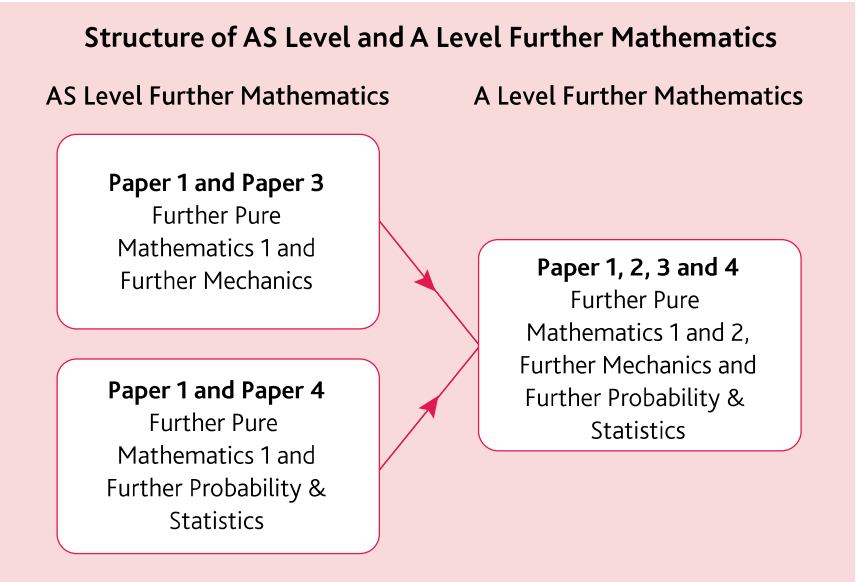
- Further Pure Mathematics 1 and Further Mechanics (Paper 1 and Paper 3)
  or
- Further Pure Mathematics 1 and Further Probability & Statistics (Paper 1 and Paper 4).

A Level Further Mathematics

Cambridge International A Level Further Mathematics includes all four components:

- Paper 1: Further Pure Mathematics 1
- Paper 2: Further Pure Mathematics 2
- Paper 3: Further Mechanics
- Paper 4: Further Probability & Statistics.

See page 10 for a table showing all possible assessment routes.



Assessment overview

Paper 1

Further Pure Mathematics 1 — 2 hours

75 marks

6 to 8 structured questions based on the Further Pure Mathematics 1 subject content

Answer all questions

Written examination

Externally assessed

60% of the AS Level

30% of the A Level

Compulsory for AS Level and A Level

Paper 2

Further Pure Mathematics 2 — 2 hours

75 marks

7 to 9 structured questions based on the Further Pure Mathematics 2 subject content

Answer all questions

Written examination

Externally assessed

30% of the A Level only

Compulsory for A Level

Paper 3

Further Mechanics — 1 hour 30 minutes

50 marks

5 to 7 structured questions based on the Further Mechanics subject content

Answer all questions

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Paper 4

Further Probability & Statistics — 1 hour 30 minutes

50 marks

5 to 7 structured questions based on the Further Probability & Statistics subject content

Answer all questions

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Recreational Problem Set #1

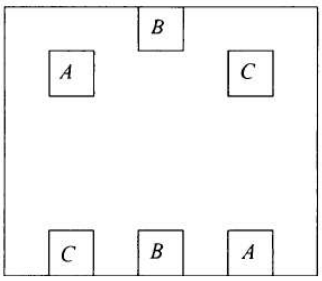
1. The Pill Problem: For 10 days, you must take exactly 1 A and 1 B pill at noon, or you will die. The pills are indistinguishable! All goes well until day 3. On this day, you shake 1 A and 2 B pills into your hand and do not know which is which. Can you survive? If so, how?

2. The Subway Love-Triangle Problem: Anna lives near a subway station located at the middle of the line. She has 2 boyfriends, Bert and Curt, who live at either end of the subway line. The men demand that she choose between them. She proposes to randomly show up at her station every day for a month and take the first train that comes. Whichever boyfriend she visits the most will be the one she chooses. The trains come every 20 minutes in each direction, every day, 24 hours per day. Is Anna's scheme fair? (You can assume that the month has 31 days, so there is no possibility of a tie.)

3. You are locked in a 50 × 50 × 50 foot room that sits on 100-foot stilts. There is an open window at the corner of the room, near the floor, with a strong hook cemented into the floor by the window. So if you had a 100-foot rope, you could tie one end to the hook and climb down the rope to freedom. (The stilts are not accessible from the window.) There are two 50-foot lengths of rope, each cemented into the ceiling, about 1 foot apart, near the center of the ceiling. You are a strong, agile rope climber, good at tying knots, and you have a sharp knife. You have no other tools (not even clothes). The rope is strong enough to hold your weight, but not if it is cut lengthwise. You can survive a fall of no more than 10 feet. How do you get out alive?

4.

Consider the following diagram. Can you connect each small box on the top with its same-letter mate on the bottom with paths that do not cross one another, nor leave the boundaries of the large box?



Recreational Problem Set #1

5.

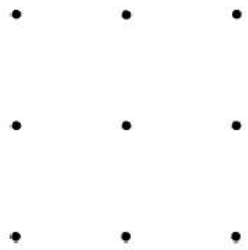
A monk climbs a mountain. He starts at 8AM and reaches the summit at noon. He spends the night on the summit. The next morning, he leaves the summit at 8AM and descends by the same route that he used the day before, reaching the bottom at noon. Prove that there is a time between 8AM and noon at which the monk was at exactly the same spot on the mountain on both days. (Notice that we do not specify anything about the speed that the monk travels. For example, he could race at 1000 miles per hour for the first few minutes, then sit still for hours, then travel backward, etc. Nor does the monk have to travel at the same speeds going up as going down.)

6.

Pat wants to take a 1.5-meter-long sword onto a train, but the conductor won't allow it as carry-on luggage. And the baggage person won't take any item whose greatest dimension exceeds 1 meter. What should Pat do?

7.

Connect all nine points below with an unbroken path of four straight lines.



8.

Lockers in a row are numbered $1, 2, 3, \ldots, 1000$. At first, all the lockers are closed. A person walks by and opens every other locker, starting with locker #2. Thus lockers $2, 4, 6, \ldots, 998, 1000$ are open. Another person walks by, and changes the "state" (i.e., closes a locker if it is open, opens a locker if it is closed) of every third locker, starting with locker #3. Then another person changes the state of every fourth locker, starting with #4, etc. This process continues until no more lockers can be altered. Which lockers will be closed?